

MATH 141

$$f(x) = \sqrt{36-x^2} \quad g(x) = \frac{x^8 - x^7 - 5x^4 + 23x^3 - x - 4}{x^2 - x - 6}$$

Solution

The number x is in the domain of $f(g(x))$ when
(1) x is in the domain of $g(x)$ and (2) $g(x)$ is in the Domain of f .

Now, x is in the Domain of $f(x) = \sqrt{x^2 - 36}$ when
 $x^2 - 36 \geq 0$ or when $x^2 \geq 36$

Taking the mtr to both sides, we find that
 $w(x) = \sqrt{x}$ is an increasing function and therefore

$$\sqrt{x^2} \geq \sqrt{36}$$

$$|x| \geq 6$$

So the Domain of f is $f_g = \{x \leq -6 \text{ or } x \geq 6\}$

Also x is in the Domain of $g(x) = \frac{x^8 - x^7 - 5x^4 + 23x^3 - x - 4}{x^2 - x - 6}$

$$\frac{x^8 - x^7 - 5x^4 + 23x^3 - x - 4}{x^2 - x - 6} = 0$$

$$\text{Domain of } g(x) = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$$

or

$$-2 < x < 3 \quad \text{Taking } x \geq 2 \quad \text{we have.}$$

$$\sqrt{x^2 - 36} \leq 2$$

and since the function $w(x) = x^2$ is increasing for $x \geq 0$
and to the function $\frac{x^8 - x^7 - 5x^4 + 23x^3 - x - 4}{x^2 - x - 6} \geq 0$

$$x^2 - 36 \leq 4$$

and

$$x^2 - 36 \leq 4 = x^2 \leq 40$$

$$|x| \leq \sqrt{40}$$

and $-\sqrt{40} \leq x \leq \sqrt{40}$

$$D_m = D_g \cap \{x \mid -\sqrt{40} \leq x \leq \sqrt{40}\}$$

$$= \{x \leq -6 \text{ or } x \geq 6\} \cap \{x \mid -\sqrt{40} \leq x \leq \sqrt{40}\}$$

$$= \{x \mid -\sqrt{40} \leq x \leq -6 \text{ or } 6 \leq x \leq \sqrt{40}\}$$